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Moments on a Coning Projectile by a Spinning Liquid in Porous Media

by Gene R. Cooper

ARL-RP-108 **September 2005**

A reprint from the *Proceedings of the Atmospheric Flight Mechanics Conference*, San Francisco, CA, 15–18 August 2005.

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Moments are predicted due to an inviscid liquid payload flowing in a sequence of end to uniform cylinders stacked in columns displaced off the spin axis of a coning projectile during free flight. These moments are then compared to moments generated by flow of the same liquid that saturates porous media contained in a sequence of uniform cylinders along the projectile symmetry axis. A modification to the classical Stewartson theory and is added to describe inertial waves in porous media. This theory is used to analysis the inertial waves produced in the liquid by the projectile coning motion. The resulting side moments are examined in reference to the type of porous media used and the assumed cylinder geometry subjected to applied coning frequencies. Eigen-frequencies and their impact on liquid moments are discussed concerning the stability of the free flight projectile.

Nomenclature

A Cylinder radius

C Aspect ratio $C = \frac{H}{D}$

$\frac{C_T}{C_x} = \frac{\text{Radial,azimuthalporoucitycoefficent}}{\text{Axilporouscitycoefficient}}$

D Cylinder Diameter

H Cylinder Length

N Number of sub-cylinders per candle-stick

P Projectile angular velocity along X' axis

S Phase factor $S = (\varepsilon + i)T$

T Non-dimensional coning frequency $T\,P = \text{Frequency}$

$\begin{bmatrix} u \\ v \\ w \end{bmatrix}$ Non-dimensional fluid velocity $\begin{bmatrix} u \\ v \\ w \end{bmatrix} A\,P^2 = \begin{bmatrix} \text{axial velocity} \\ \text{radial velocity} \\ \text{azimuthal velocity} \end{bmatrix}$

$A\,R_0$ Displacement of candle-stick symmetry axis

V_0 Forward velocity of projectile

$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} =$ Inertial coordinate system

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$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \quad \text{Body fixed coordinate system}$$

Δ Sub-cylinder aspect ratio $\Delta = 2C/N$

ε Damping rate of spiral motion

ρ Fluid mass density

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \quad \text{Euler roll, pitch and yaw angles of the projectile}$$

I. Introduction

Predicting the moment due to a liquid payload in a spinning and coning projectile is a problem of considerable interest to Army. Stewartson[1] considered the linear problem of a liquid payload in a spinning right circular cylinder using separation of variables and eighenvalue expansions for an inviscid liquid. i First order viscous boundary layer corrections to the Stewartson[1] theory were carried out by Wedemeyer[2] and Murphy.[3] A method for calculating the linear liquid moment using the full linear viscous equations with boundary layer corrections confined only to the endcanps was also presented by Hall and Sedney and Gerber[4,5].

A further interest to the army is to consider a series of uniform circular cylinders stacked end to end separated by impenetrable end-caps (candle-sticks). These candle-stick(s) may be situated along the symmetry axis or off set but parallel to the symmetry axis of the projectile. Coning motion induced liquid moments are considered here for a number of candle-stick(s) configurations. The eigen-frequencies for such configurations are shown to be identical to those found by Stewartson[1].

Liquid payloads contained in a highly permeable material have also been of interest to the Army for some time. Laboratory tests and flight tests have shown that a highly preamble medium can significantly reduce the spin-up time of a liquid payload.[6] Flight stability for liquid saturated permeable payloads has also been examined by D'Amico.[7] The work here extends the Stewartson[1] problem by considering a cylindrical cavity filled with a permeable medium that is impregnated with an invisicd liquid. A further modification is introduced by segmenting the cavity, along the symmetry axes, into a sequence of equal length cylinders. Each of these cylinders is separated by impermeable end-caps. The porous media is modeled by a drag term, which is proportional to the liquid velocity relative to the assumed ridge porous media that is added to the linearized Euler equations. This analysis examines the induced liquid moment as a function of parameters found by Stewartson1 plus parameters describing the porous media and the number of segments in the cylindrical cavity.

II. Equations of Motion for the Candle-Stick Configurations

Figure 1. shows the X', Y', Z' axes rotating uniformly about Z' with angular speed $\mathbf{P} = (P,\ 0,\ 0)$. The liquid is assumed initially to be rotating as a rigid body with the same angular speed $\mathbf{P}$ so the velocity $\mathbf{V}'$ of the liquid inside one the cylinders is

$$\mathbf{V}' = \mathbf{P} \times (x,\ R_0 \cos B + r\cos\theta,\ R_0 \sin B + r\sin\theta) \quad (1)$$

The unperturbed state for Eq. (1) satisfies the Euler equation:

$$P^2\, \mathbf{e}_x \times (\mathbf{e}_x \times (x,\ R_0 \cos B + r\cos\theta,\ R_0 \sin B + r\sin\theta)) = -\nabla \frac{P_s}{\rho} \quad (2)$$

for which P_s is the unperturbed liquid pressure. Following Stewartson* and letting $\mathbf{R} = (x,\ R_0 \cos B + r\cos\theta,\ R_0 \sin B + r\sin\theta)$ gives the scalar potential

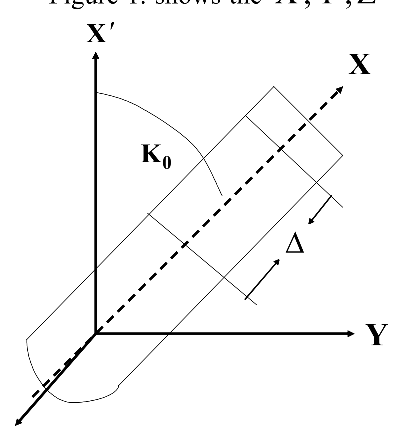

Figure 1 Coordinate Systems of configuration

$$P_S/\rho = P^2 R_0 \cos(B-\theta) - P^2 r^2/2 \tag{3}$$

Now consider a perturbed angular velocity of the projectile for small ω_y, ω_z given by

$$\mathbf{\Omega} = \mathbf{P} + (0,\ \ \omega_y,\ \ \omega_z) \tag{4}$$

and adopting projectile-fixed X, Y, Z axes causes the liquid velocity to take the form

$$\mathbf{V} = \mathbf{P} \times (x,\ \ R_0 \cos B + r\cos\theta,\ \ R_0 \sin B + r \sin\theta) + \mathbf{v} \tag{5}$$

where the components of $\mathbf{v}$ have the same order of magnitude as ω_y, ω_z.

The Euler equations for small perturbations written in the body fixed frame now read as

$$\begin{aligned} \frac{dV}{dt} - 2PW + \frac{1}{\rho}\frac{d\hat{p}}{dr} &= 0 \\ \frac{dW}{dt} + 2PV + \frac{1}{\rho r}\frac{d\hat{p}}{d\theta} &= 0 \\ \frac{dU}{dt} + \frac{1}{\rho}\frac{d\hat{p}}{dx} &= 0 \end{aligned} \tag{6}$$

where $(V,\ \ W,\ \ U)$ are the cylindrical components of the perturbed velocity, $\hat{p}$ is the perturbed pressure.

The boundary conditions at the solid wall satisfy

$$(V,\ \ W,\ \ U) \bullet \mathbf{n} = \mathbf{\Omega} \times \mathbf{R}s \bullet \mathbf{n} \tag{7}$$

where $\mathbf{n}$ is a outward unit vector on the wall and $\mathbf{R}$s a point on a cylinders wall. Following Murphy/Cooper* the perturbation quantities ω_y, ω_z are assumed to represent coning motion given by

$$\begin{aligned} \omega_y &= \ \ K_0 P\left(\varepsilon T \sin(PTt) + (T-1)\cos(PTt)\right)e^{\varepsilon PTt} \\ \omega_z &= -K_0 P\left((T-1)\sin(PTt) - \varepsilon T \cos(PTt)\right)e^{\varepsilon PTt} \end{aligned} \tag{8}$$

where the coning damping rate is ε the coning frequency is T and K_0 is the magnitude of the coning angle. Equation (8) is now used in Eq. (7) so the boundary conditions now become:

$$\begin{aligned} \hat{V} &= \Re e\left(x\, K_0 P\,(S-i)\, e^{PSt+i\theta}\right) \\ \hat{U} &= \Re e\left(K_0 P\left(e^{iB} R_0 + r e^{i\theta}\right)(S-i)\, e^{PSt+i\theta}\right). \\ i &= \sqrt{-1},\ S \equiv (\varepsilon + i)T \end{aligned} \tag{9}$$

These boundary conditions suggest Eq. (6) is separable giving

$$\begin{bmatrix} V \\ W \\ U \\ \hat{p} \end{bmatrix} = \begin{bmatrix} v \\ w \\ u \\ p \end{bmatrix} e^{PSt} \tag{10}$$

and solving for the velocity components yields:

$$v = \frac{\frac{dp}{dr} rS + 2\frac{dp}{d\theta}}{r\rho P\left(S^2+4\right)}$$
$$w = \frac{\frac{dp}{d\theta} S - 2r\frac{dp}{dr}}{r\rho P\left(S^2+4\right)} \tag{11}$$
$$u = \frac{\frac{dp}{dx}}{\rho P S}$$

Using the continuity equation, $\nabla(v,\ w,\ u) = 0$, produces the following equation for the pressure p

$$r^2\frac{d^2p}{dr^2} + r\frac{dp}{dr} + \frac{d^2p}{d\theta^2} - r^2\sigma^2\frac{d^2p}{dx^2} = 0$$
$$\sigma^2 = -\frac{S^2+4}{S^2} \tag{12}$$

At this point in the analysis it is useful to consider each cylindrical candle-stick ranging from, $-C \le x \le C$, to consist of a end to end sequence of N equal length, Δ, cylinders with impentetrible end caps such that $\Delta = 2C/N$. Applying Eq. (9) to each of the sub-cylinders shows that separation of variables gives

$$\begin{aligned} p = {} & A_j K_0 P^2 a^2 \rho \cos\left(\pi(2j+1)\right)(C+x)/\Delta)J_1\left(\pi(2j+1)\right)\sigma r/\Delta)e^{i\theta} \\ & + K_0 P^2 a^2 \rho\left(r\,x\,E/a^2 + r\,D/a\right)e^{i\theta} + K_0 P^2 a \rho F x \end{aligned} \tag{13}$$

where

$$\begin{aligned} A_j &= \frac{8(-1)^n \Delta^2 (S-i)^2 (S+2i)}{\begin{pmatrix} j_1(\pi(2j+1)\sigma a/\Delta)\Delta(S-2i) - \\ \pi a(2j+1)S\sigma j_0(\pi(2j+1)\sigma a/\Delta) \end{pmatrix}\pi^2 a^2 (2j+1)^2} \\ E &= (S-i)S \\ D &= ((2n+1)\Delta - 2C)(S-i)^2 \\ F &= e^{iB} R_0 (S-i)S/a \\ 0 &\le n \le N,\ j = 0,1,2\cdots \end{aligned} \tag{14}$$

III. Candle-Stick(s) Liquid Moments

The moment induced by the liquid contained in the segmented cavity is calculated from the time derivative of the angular momentum field. Non-dimensionalizing the moment with $2\pi\rho a^4 C P^2$ it is convenient to write the side moment components as[3]

$$\begin{aligned} M_Y + iM_Z &= \tau C_{LM}\left(2\pi\rho a^4 C P^2\right)K_1 e^{PTt} \\ C_{LM} &= C_{LSM}(T,N,C) + iC_{LIM}(T,N,C) \end{aligned} \tag{15}$$

Unit vectors of the body-fixed cylindrical coordinates, $(\mathbf{e_X},\ \mathbf{e_r},\ \mathbf{e_\theta})$ are written as the complex quantities in terms of $(\mathbf{e_X},\ \mathbf{e_Y},\ \mathbf{e_Z})$

$$\begin{aligned}\mathbf{e_r} &= \mathbf{e_Y}\cos\theta + \mathbf{e_Z}\sin\theta \Leftrightarrow \exp(i\theta)\\ \mathbf{e_\theta} &= -\mathbf{e_Y}\sin\theta + \mathbf{e_Z}\cos\theta \Leftrightarrow i\exp(i\theta)\end{aligned} \tag{16}$$

Placing these in the moment integral and using the Reynolds Transport Theorm[10] yields the following expression for the liquid moment coefficient:

$$\begin{aligned}TC_{LM} &= \frac{1}{2\pi\frac{C}{a}}\sum_{n=0}^{N}\iint_{\partial}\left\{(x\,\mathbf{e_X} + r\,\mathbf{e_r})\times\left[(S-i)\mathbf{q} + 2\,\mathbf{e}_X\times\mathbf{q}\right] - i\left[r^2 - 2\,x\right]e_X\right\}r\,dr\,dx\\ \mathbf{q} &= (v,\ \ w,\ \ u)\end{aligned} \tag{17}$$

The last result written terms of Eq.(11) is :

$$\begin{aligned}TC_{LM} &= \frac{128\,i\,C^3}{\pi^4 N^2}(S-i)^2(S+2i)\left(\frac{S-2i}{\sigma^2 S} - \frac{2}{\sigma^2} + 1\right)\\ &\sum_j \frac{j_1(zz)/j_0(z)}{\left((\pi(2j+1)\sigma NS) - \frac{2C(S-2i)j_1(zz)}{j_0(zz)}\right)}\\ &+ \frac{\left(\begin{array}{l} i(S-i)\left((4C^2+3)N^2 - 16C^2\right)S^2 + 16C^2(1-N^2) - \\ 2i\left(16C^2 - N^2(3+8C^2)\right)\end{array}\right)}{12N^2(S-2i)}\\ &+ iR_0^2 S(S-i)\sin B\, e^{iB}\ ;zz \equiv \pi(2j+1)\sigma N/2C\end{aligned} \tag{14}$$

For σ not a resonance value and as the number of cylinders becomes large ($N\to\infty$) the value of C_{LM} approaches the frozen liquid limit given by:

$$\begin{aligned}C_{LM} &\to -i\,e^{-iB}(2\varepsilon\sin B + \cos B)R_0^2\,T - \frac{(4C^2-3)(2\varepsilon+i)}{12}T\\ &+ i\,e^{-iB}(2\varepsilon\sin B + \cos B)R_0^2 + \frac{(4C^2+1)(\varepsilon+i)}{4}\end{aligned} \tag{15}$$

The eigen-values for this problem can be found by inspecting Eq. (14) and after some algebra these are found to be zeros of

$$\begin{aligned}\frac{J_1(zz)}{zzJ_0(zz)} &= \frac{\pm\sqrt{B+1}-1}{B}\\ B &\equiv \frac{4zz^2C^2}{\pi^2(2j+1)^2N^2}\end{aligned} \tag{16}$$

which gives the Stewartson[1] values when zz is solved for frequency T.

IV. Equations of Motion for the Axial Porous Media Configuration

For this problem the moment arm $R_0 = 0$ in Eqs. (3 & 5) and the position vector becomes $\mathbf{R} = (x,\ \ r\cos\theta,\ \ r\sin\theta)$ since the candle-stick axis is the symmetry axis of the projectile. However, the Euler equations are now modified to account for an inviscid liquid flowing through porous media. The modification is assumed to have terms proportional to the liquids velocity relative to that of the porous media which is taken to have the same velocity as the coning projectile. The Euler equations are now written as:

$$\frac{dV}{dt} - 2PW + C_t P\left(V - \hat{V}\right) + \frac{1}{\rho}\frac{d\hat{p}}{dr} = 0$$
$$\frac{dW}{dt} + 2PV + C_t P\left(W - \hat{W}\right) + \frac{1}{\rho r}\frac{d\hat{p}}{d\theta} = 0 \tag{17}$$
$$\frac{dU}{dt} + C_x P\left(U - \hat{U}\right) + \frac{1}{\rho}\frac{d\hat{p}}{dx} = 0$$

where the velocity of the media obtained from the rotation kinematics is given by

$$\hat{V} = xK_0 P(S - i)e^{PSt + i\theta}$$
$$\hat{W} = ixK0P(S - i)e^{PSt + i\theta} \tag{18}$$
$$\hat{U} = -rK_0 P(S - i)e^{PSt + i\theta}$$

Separating variables according to

$$\begin{bmatrix} V \\ W \\ U \\ \hat{p} \end{bmatrix} = \begin{bmatrix} v \\ w \\ u \\ p \end{bmatrix} e^{PSt + i\theta} \tag{19}$$

leads to the solution of Eq. (16)

$$v = -\frac{2ip + \frac{dp}{dr}(S + C_t)}{r\rho P\left((S + C_t)^2 + 4\right)} + \frac{xC_t P(S - i)}{S + C_t - 2i}$$
$$w = -\frac{i(S + C_t)p - 2\frac{dp}{dr}}{r\rho P\left((S + C_t)^2 + 4\right)} + \frac{ixC_t P(S - i)}{S + C_t - 2i} \tag{20}$$
$$u = -\frac{\frac{1}{\rho P}\frac{dp}{dx} + rC_x P(S - i)}{S + C_x}$$

Continuing with the continuity equation gives the following equation for the perturbation pressure p

$$r^2\frac{d^2p}{dr^2} + r\frac{dp}{dr} - p - r^2\bar{\sigma}^2\frac{d^2p}{dx^2} = 0$$
$$\bar{\sigma}^2 = -\frac{(S + C_t)^2 + 4}{(S + C_t)(S + C_x)} \tag{21}$$

Solving and using the demands of Eq. (17) produces the following coefficients for expression Eq. (13)

$$
\begin{aligned}
A_j &= \frac{8(-1)^n \Delta^2 (S-i)^2 (S+C_t+2i)}{\begin{pmatrix} j_1(\pi(2j+1)\overline{\sigma}a/\Delta)\Delta(S+C_t-2i)- \\ \pi(2j+1)(S+C_t)\overline{\sigma}\, j_0(\pi(2j+1)\overline{\sigma}a/\Delta) \end{pmatrix}\pi^2 (2j+1)^2} \\
E &= (S-i)S \\
D &= -((2n+1)\Delta - 2C)(S-i)^2 \\
F &= 0
\end{aligned}
\tag{22}
$$

$$
0 \le n \le N,\ j = 0,1,2\cdots
$$

V. Porous Media Liquid Moments

Using the above procedure for calculating liquid side moments results in the following expression for the porous media moments

$$
\begin{aligned}
TC_{LM} = &-\frac{128iC^3}{\pi^4\overline{\sigma}^2 N^2 (S+C_t)(S+C_x)}(S-i)^2(S+Ct+2i) \\
&\begin{pmatrix} 2S^3 + 2(C_x+3C_t-i)S^2 + 2(2C_tC_x + 3C_t^2 - 2iC_t + 4)S \\ 2(C_t+C_x-i)(C_t^2+4) \end{pmatrix} \\
&\sum_j J_1(zz)/(2j+1)^4 \begin{pmatrix} 2J_1(zz)(S+C_t)(S+C_t-4i)C- \\ \pi(2j+1)\sigma N\, J_0(zz)(S+C_t)(S+C_t-2i) \end{pmatrix} + \\
&iC^2S((N^2-2)S - i(3N^2-4))/3N^2 - \frac{2iC^2(S-i)^2(S+2C_x+C_t)}{3N^2(S+C_t-2i)} + \\
&iS(S-i)/4 - 2iC^2(2Cx+Ct)/3N^2(S-2i); \\
zz \equiv\ &\pi(2j+1)\overline{\sigma}N/2C
\end{aligned}
\tag{23}
$$

Here again the limiting case with $N \to \infty$ gives the value of C_{LM} for the frozen liquid side moment

$$
TC_{LM} \to iS((4C^2+3)S - 3i(4C^2+1))/12 \tag{24}
$$

The eigen-values for this problem are found from Eq. (22) and after some algebra they are zeros of

$$
\frac{J_1(zz)}{zzJ_0(zz)} = \frac{\pm(C_x - C_t + 2i)\sqrt{B((C_x-C_t)^2B - 16) - 16}}{2((C_x-C_t)(C_x-C_t+4i) - 16)B} + \frac{(C_x-C_t)B - 4i}{2(C_x-C_t+2i)B} \tag{25}
$$

This equation shows when $C_x = C_t$ yields eigen-values $\overline{zz}$ are the same as those found by Stewartson[1],Eq. (16), provided $zz = \overline{zz}$.

VI. Calculation Method

The equations of the last sections need to be calculated for a wide range of flight and porous media parameters all of which need the values of Bessel functions. For small values of $|zz|$ simply using power series expansions of each Bessel function works very well. Bessel functions at large values of $|zz|$ were obtained by using asymptotic expansions for each Bessel funcion[9]. Generally calculating Bessel functions for complex arguments for intermediate values of $|zz|$ is a non-trivial problem and the methods used here employs Gauss continued fractions. This author has judged that a further discussion of these methods is not appropriate for this article but the reader should be aware of the numerical difficulties associated with calculating Bessel functions.

VII. Results

Figures 2 and 3 show the side moment C_{LSM} and in plane moment C_{LIM} as functions of the non-dimensional frequency $\mathbf{T}$. These moments show the eigen-frequencies for a candle stick that is displaced off the symmetry axis by $R_0 = 1.5$.

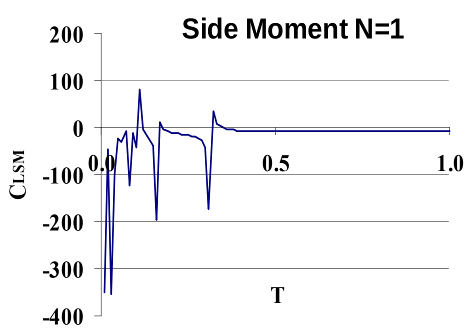

Figure 2 Liquid Side Moment

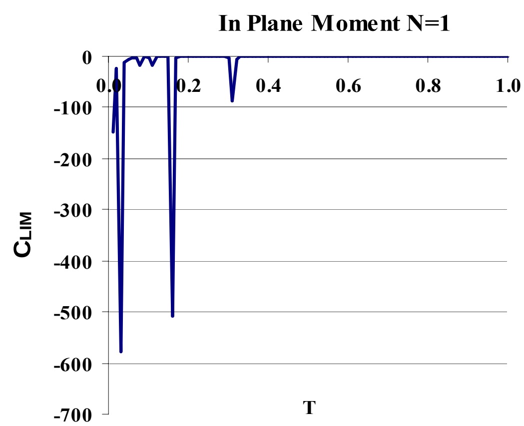

Figure 3 Liquid In-Plane Moment

The next two plots given in Fig. 4 & 5 present the liquid moments for three candle sticks uniformly distributed around the projectile symmetry axis for $R_0 = 1.5$. The value of ε and N are chosen so that Stewartson[1] eigen-values are not present for the given range of T. Figure 5 also displays the frozen liquid moment which indicates that the liquid behaves like frozen liquid for increasing values on N provided eigen-frequencies are avoided.

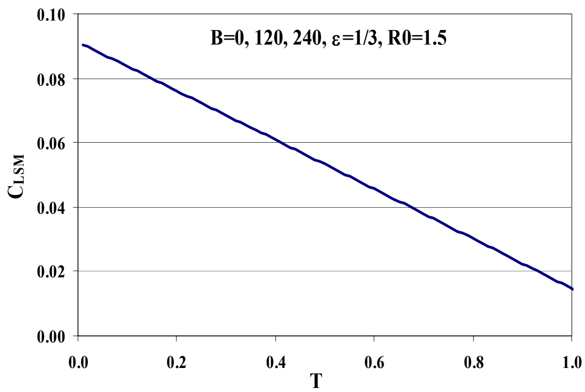

Figure 4 Sum Of Three Side Plane Moments

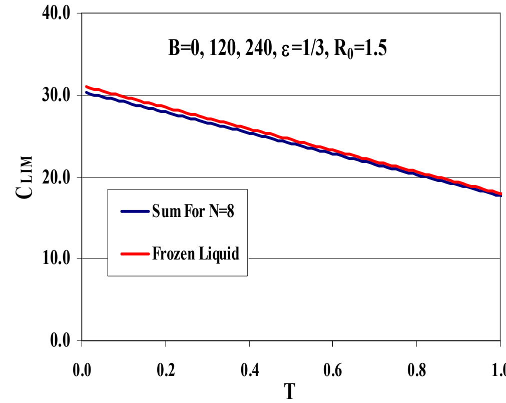

Figure 5 Sum of Three In Plane Moments

Next consider a centrally located candle stick containing a liquid in porous media. Figures 6 and 7 presents examples of liquid moments for typical values of $C_T = 0.3,$, $C_x = 0.3, 0.5$ and $\varepsilon = -0.612, -1.807$. Once again the moments display the eigen-frequency behavior of saturated porous media with increasing $N = 1$. These indicate that porous media forces the eigen-frequencies to be complex when ever ε is chosen to cause $\overline{\sigma}$ to be an eigen-value

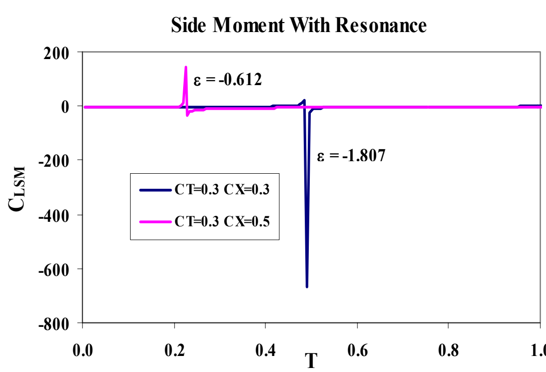

Figure 6 Side Moment For Central Porous Cavities

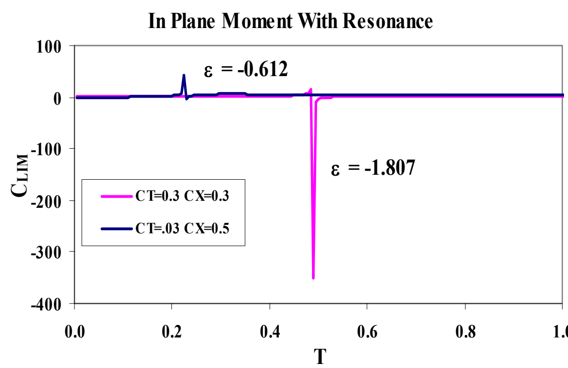

Figure 7 In-Plane Moments For Central Porous Cavities

VIII. Conclusions

The off axis candle stick problem has been shown to be equivalent to the inviscid Stewartson[1] problem whenever porous media is not present or can be ignored. Resonances are independent of the candle stick off axis position.

In the case of a candle stick, containing saturated porous media, located along the symmetry axis of the projectile generally forces resonances only for complex values of S. In all cases the liquid moments approach the values for a frozen liquid when eigen-frequencies are not present. In the particular case where $C_T = C_X$ it is possible find the resonances from the Stewartson[1] tables but if $C_T \neq C_X$ requires a numerical search for resonances in the complex plane for $\overline{zz}$ from which S can then be calculated.

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